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Graphing Rational Functions

Unit 11 Lesson 6

Graphing Rational Functions

Students will be able to:

Compute for the functional values given the domain and sketch the graph of a rational function through Cartesian coordinate system.

Key Vocabulary:

- Cartesian Coordinate System
- Domain
- Range
- Function

Function:

The function is a relation such that no two ordered pairs have the same first element. A function may be denoted as $y = f(x)$ which is read "f of x". A function may be written as $f: x \rightarrow y$, where $x \in \text{domain}$ while $y \in \text{range}$.

Graphing Rational Functions

Sample Problem 1: Find the Range of the following rational function.

1. If $f(b) = \frac{b-b^2}{1+b^2}$, find a. $f\left(-\frac{1}{2}\right)$; b. $f(-2)$; c. $f(-1)$.

$$\begin{aligned} \text{a. } \frac{\left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right)^2}{1 + \left(-\frac{1}{2}\right)^2} &= \frac{-\frac{3}{4}}{\frac{5}{4}} = -\frac{3}{4} \times \frac{4}{5} = -\frac{12}{20} = -\frac{3}{5} \end{aligned}$$

Graphing Rational Functions

Sample Problem 1: Find the Range of the following rational function.

1. If $f(b) = \frac{b-b^2}{1+b^2}$, find a. $f\left(-\frac{1}{2}\right)$; b. $f(-2)$; c. $f(-1)$.

$$b. \frac{(-2) - (-2)^2}{1 + (-2)^2} = \frac{-2 - 4}{1 + 4} = \frac{-6}{5} = -1\frac{1}{5}$$

Graphing Rational Functions

Sample Problem 1: Find the Range of the following rational function.

1. If $f(b) = \frac{b-b^2}{1+b^2}$, find a. $f\left(-\frac{1}{2}\right)$; b. $f(-2)$; c. $f(-1)$.

$$c. \frac{(-1) - (-1)^2}{1 + (-1)^2} = \frac{-1 - 1}{1 + 1} = \frac{-2}{2} = -1$$

Graphing Rational Functions

Sample Problem 2: Draw the graph of the following rational function.

2. Below is the table containing the domain of $f(x) = x/2$, find the range and draw graph its function.

X	-2	-1	0	1	2	3
Y	-1	$-1/2$	0	$1/2$	1	$3/2$

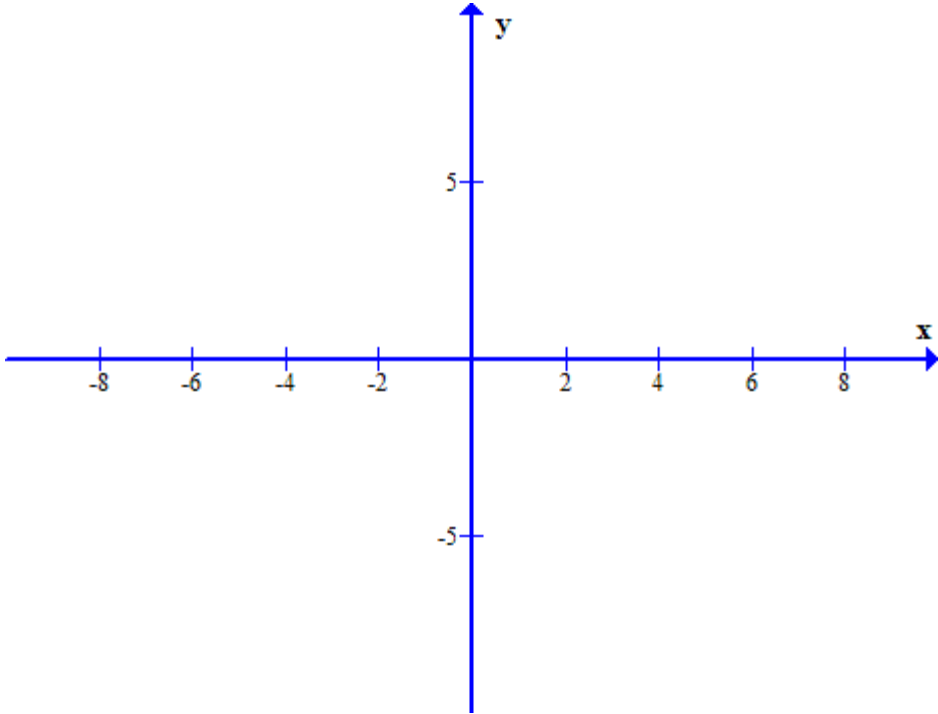
$$f(-2) = \frac{-2}{2} = -1 \quad f(-1) = \frac{-1}{2} \quad f(0) = \frac{0}{2} = 0$$

$$f(1) = \frac{1}{2} \quad f(2) = \frac{2}{2} = 1 \quad f(3) = \frac{3}{2}$$

Graphing Rational Functions

Sample Problem 2: Draw the graph of the following rational function.

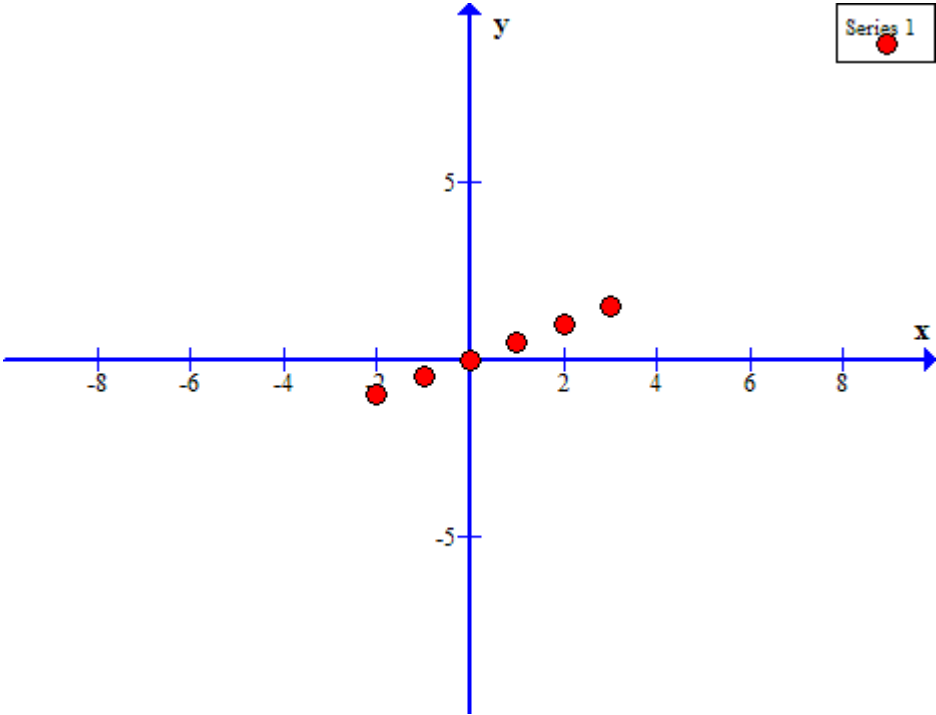
X	-2	-1	0	1	2	3
Y	-1	-1/2	0	1/2	1	3/2



Graphing Rational Functions

Sample Problem 2: Draw the graph of the following rational function.

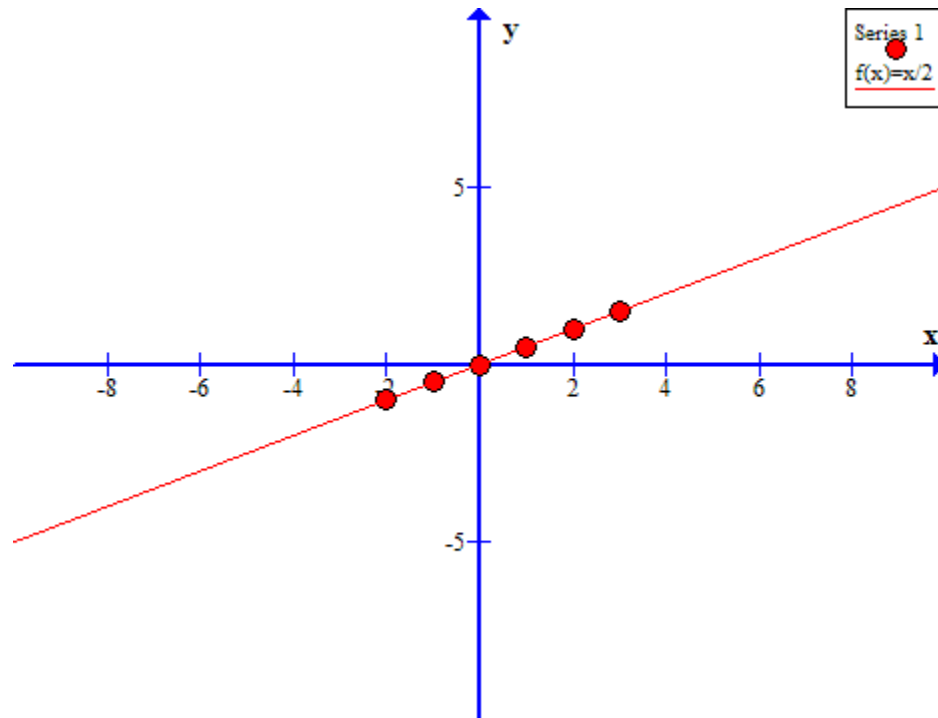
X	-2	-1	0	1	2	3
Y	-1	-1/2	0	1/2	1	3/2



Graphing Rational Functions

Sample Problem 2: Draw the graph of the following rational function.

X	-2	-1	0	1	2	3
Y	-1	-1/2	0	1/2	1	3/2



Graphing Rational Functions

Sample Problem 2: Draw the graph of the following rational function.

3. Below is the table containing the domain of $f(x)$, find the range and draw graph its function.

X	-2	-1	0	1	2	3
Y	-3	$-3/2$	0	$3/2$	3	$9/2$

$$f(-2) = \frac{3(-2)}{2} = -3$$

$$f(-1) = \frac{3(-1)}{2} = -\frac{3}{2}$$

$$f(0) = \frac{3(0)}{2} = 0$$

$$f(1) = \frac{3(1)}{2} = \frac{3}{2}$$

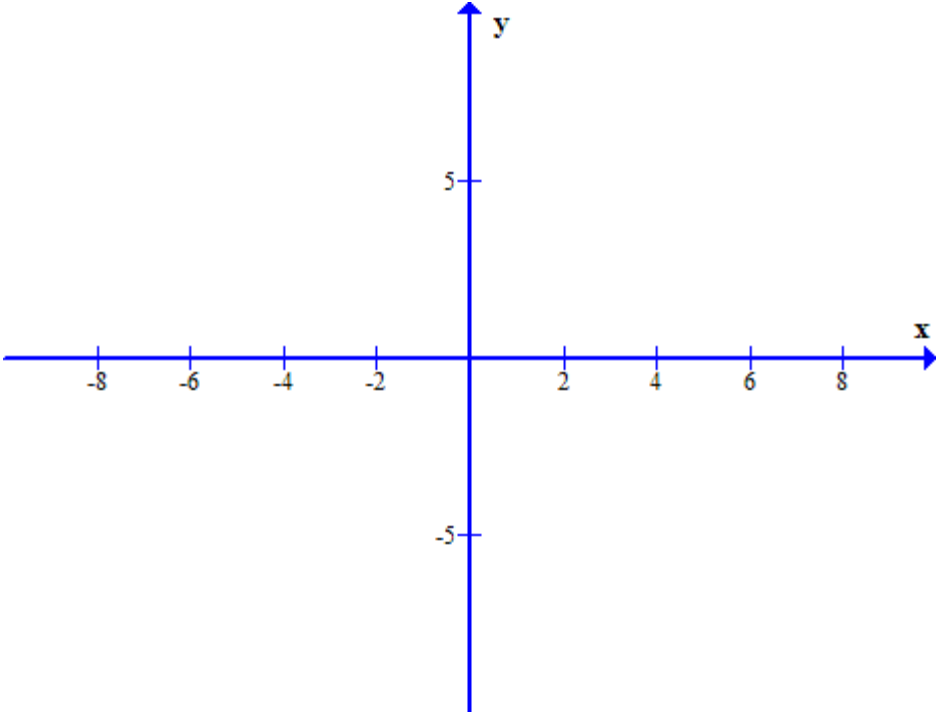
$$f(2) = \frac{3(2)}{2} = 3$$

$$f(3) = \frac{3(3)}{2} = \frac{9}{2}$$

Graphing Rational Functions

Sample Problem 2: Draw the graph of the following rational function.

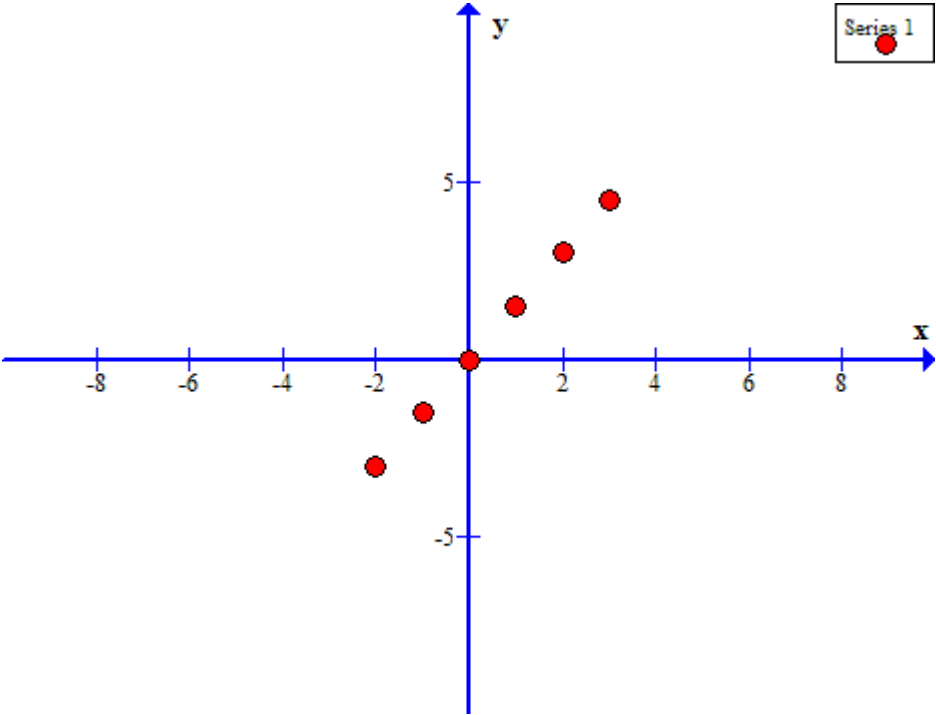
X	-2	-1	0	1	2	3
Y	-3	-3/2	0	3/2	3	9/2



Graphing Rational Functions

Sample Problem 2: Draw the graph of the following rational function.

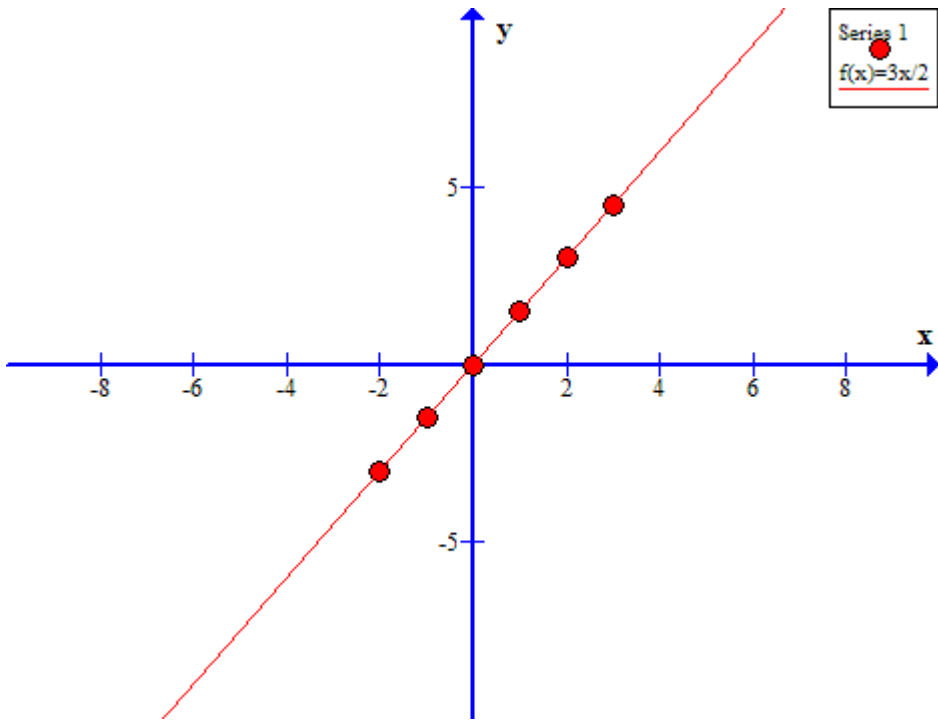
X	-2	-1	0	1	2	3
Y	-3	-3/2	0	3/2	3	9/2



Graphing Rational Functions

Sample Problem 2: Draw the graph of the following rational function.

X	-2	-1	0	1	2	3
Y	-3	-3/2	0	3/2	3	9/2



Graphing Rational Functions

Sample Problem 2: Draw the graph of the following rational function.

4. Find the graph of $y = \frac{x+1}{-2}$, using x and y intercept.

Solution:

If $x = 0$

$$y = \frac{x+1}{-2} = \frac{0+1}{-2} = -\frac{1}{2} \quad \left(0, -\frac{1}{2}\right)$$

If $y=0$

$$y = \frac{x+1}{-2} \quad (-2)y = x+1 \quad -2y-1 = x \quad -2(0)-1 = x \quad x = -1 \quad (-1, 0)$$

Graphing Rational Functions

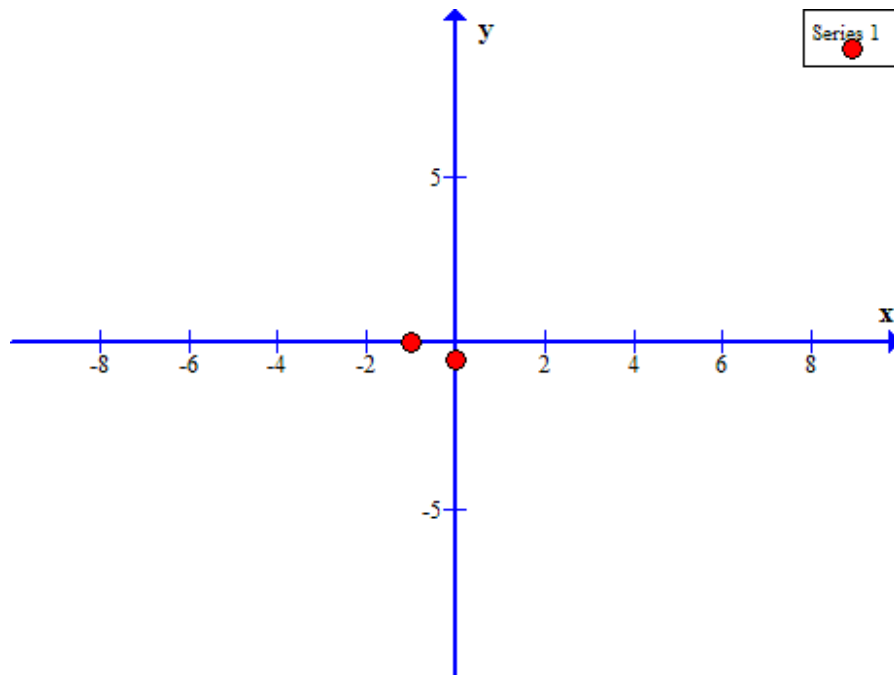
Sample Problem 2: Draw the graph of the following rational function.

4. Find the graph of $y = \frac{x+1}{-2}$, using x and y intercept.

Solution:

$$\left(0, -\frac{1}{2}\right)$$

$$(-1, 0)$$



Graphing Rational Functions

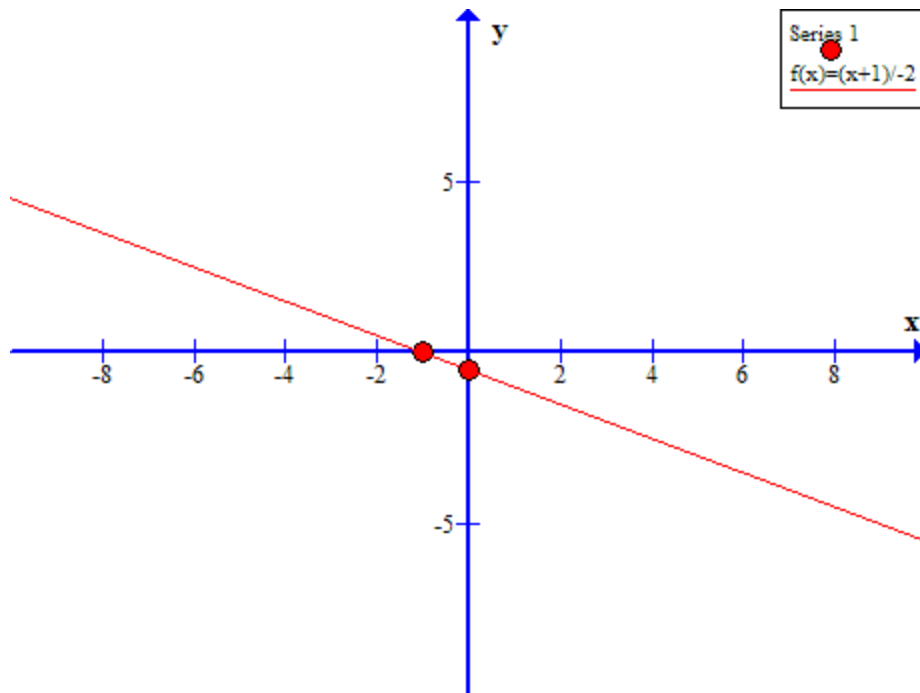
Sample Problem 2: Draw the graph of the following rational function.

4. Find the graph of $y = \frac{x+1}{-2}$, using x and y intercept.

Solution:

$$\left(0, -\frac{1}{2}\right)$$

$$(-1, 0)$$



Graphing Rational Functions

Graph of Rational Function

The line $x = a$ is a vertical asymptote if the graph increases or decreases without bound on one or both side of the line as x closer to closer to $x = a$.

The line $y = b$ is a horizontal asymptote if the graph approaches $y = b$ as x increases or decreases without bound. Note that it doesn't have to approach $y = b$ as both increases and decreases. it only need to approach it on one side in order for it to be a horizontal asymptote.

Graphing Rational Functions

Graph of Rational Function

1. The graph will have a vertical asymptote at if the denominator is zero at $x=a$ and the numerator isn't zero at $x=a$.
2. If $n < m$ then x axis is the horizontal asymptote.
3. If $n = m$ then the line $y = a/b$ is the horizontal asymptote.
4. If $n > m$ there will be no horizontal asymptote.

$$f(x) = \frac{ax^n + \dots}{bx^2 + \dots}$$

Sample Problem 3: Draw the graph of the following rational

4. Sketch the graph of a function $f(x) = \frac{3x+1}{2x-1}$

y – Intercept

$$f(0) = \frac{3(0)+1}{2(0)-1} = -1$$

x – Intercept

$$3x+1=0 \quad x = -\frac{1}{3}$$

Vertical Asymptote

$$2x-1=0 \quad x = \frac{1}{2}$$

Horizontal Asymptote

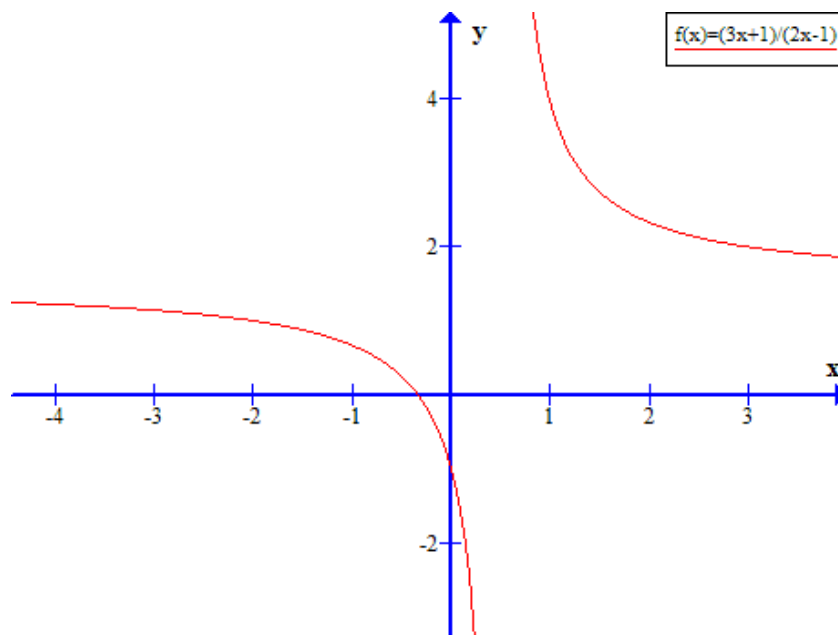
$$a=3, b=2$$

$$y = \frac{a}{b} = \frac{3}{2}$$

Graphing Rational Functions

Sample Problem 3: Draw the graph of the following rational

4. Sketch the graph of a function $f(x) = \frac{3x+1}{2x-1}$



Graphing Rational Functions

Sample Problem 3: Identify the function and the range of the following graph.

5. Find the domain and range of the graph of $f(x) = \frac{x^2}{2}$.

$$\underline{f(x) = (x)(x)/2}$$

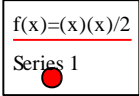


Graphing Rational Functions

Sample Problem 3: Identify the function and the range of the following graph.

5. Find the domain and range of the graph below.

Identify the Points



●(-3,4.5)

● (3,4.5)

●(-2,2)

●(2,2)

(-1,0.5)●

(0,0)
●

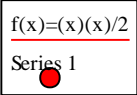
● (1,0.5)



Graphing Rational Functions

Sample Problem 3: Identify the function and the range of the following graph.

5. Find the domain and range of the graph below.



Ordered pair :
 $\{(-3, 9/2), (-2, 2), (-1, 1/2), (0, 0), (1, 1/2), (2, 2), (3, 9/2)\}$

$(-3, 4.5)$

$(3, 4.5)$

Domain: $\{.....-3, -2, -1, 0, 1, 2, 3 ...\}$
all positive and negative real numbers
Range: $\{0, 1/2, 2, 9/2 ...\}$
all positive real numbers

$(-2, 2)$

$(2, 2)$

$(-1, 0.5)$

$(0, 0)$

$(1, 0.5)$